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Monetary Uncertainty and Default

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Abstract

We investigate the effects of monetary uncertainty on the aggregate economy, especially default. First, we estimate monetary policy for the U.S. that allows for time-varying volatility to bring in monetary uncertainty. Then, we assign productivity, money policy and monetary uncertainty shock to a dynamic general equilibrium model with default that is calibrated with the U.S. economy. It reveals that monetary uncertainty has a negative effect on the economic activity and results in default issue. An increase of risk aversion among agents is the primary cause of investment delays and dries up liquidity temporarily while a decrease in the output serves as an intermediate step in the transmission mechanism of monetary uncertainty.

Keywords: Default, Monetary policy, Uncertainty shock

JEL Classification: E37, E52

1 Introduction

The banking sector clearly has a considerable effect on the real economy. Closer to home, the global financial crisis in late 2007 triggered a severe worldwide economic downturn. At its centre are the banks: subprime credit exposure and significant write-offs on asset-backed securities were the trigger for subsequent events. It is commonly held that financial frictions have a great influence on the business cycle, as credit markets and the real economy interact with each other. Thus, financial frictions such as money and default have a close linkage with real economy. To incorporate financial frictions into our model, as suggested by Tsomocos (2003) and Goodhart et al. (2006), we introduce two essential financial frictions: money via "cash-in-advance" (CIA) constraints and default as equilibrium phenomena. The possibility of default on any debt obligations underscores the necessity of CIA constraints. The interaction of money and default justifies fiat money as the stipulated means of exchange. Otherwise, the mere presence of a monetary sector without the possibility of default or any other financial friction in equilibrium may become a veil without affecting real trade and final equilibrium allocation. Indeed, CIA constraints are a minimal institutional arrangement to capture the fundamental aspect of liquidity and its interactions with default to affect the real economy.

Considering the role of financial frictions, policymakers have begun to highlight their importance in economic systems, beginning –at last– to treat those factors as a crucial part of their overall policy stance. However, besides the direct effect of their policy, it is still unclear whether the uncertainty of governments' policy will also have a real effect on them? For the policy uncertainty, we focus on the monetary uncertainty. As for financial frictions, we focus on money and default, which constitute an essential part of financial stability. The goal of this paper is to try to confirm the role of monetary uncertainty and clarify the mechanism through which monetary uncertainty influences default. The following are details about how we construct our model based on current literature.

As mentioned before, to set up models, we adopt CIA constraints –an idea that originated with Clower (1967)– which captures the role of money as a medium of exchange. Therefore, each agent faces a CIA constraint, in addition to their intertemporal budget constraint. This constraint is such that the agent's real spending in any given period cannot exceed the amount of real money balances the agent carried into that period. Under such a scenario, when there is an opportunity cost to hold the money (i.e. a positive nominal interest rate), then if the agent has no uncertainty about the future path of income, they will hold only the amount of money that is exactly sufficient to finance their desired level of consumption.

In order to take financial frictions into consideration, the current literature mostly rests on two different approaches. One is from the seminal paper by Kiyotaki and Moore (1997): It introduces financial frictions via collateral constraints. Agents are heterogeneous in terms of their rate of time preference, which means that they can be divided into lenders and borrowers. The financial sector intermediates between these groups and introduces frictions by requiring that borrowers provide collateral for their loans. Bernanke and Gertler (1989) is behind the second stream of research. In their paper, financial frictions are incorporated into a general equilibrium model. This approach is further developed by Carlstrom and Fuerst (1997) and then merged with the New Keynesian framework by Bernanke et al. (1999). This was the benchmark model with financial frictions used throughout the 2000s. Frictions arise in this model because monitoring of the loan applicant is costly, causing an endogenous wedge to be driven between the lending rate and the risk-free rate. This means that financial frictions affect the economy via the prices of loans. The external finance premium setup is used to analyse financial frictions in the Great Depression by Christiano et al. (2005) and to study the business cycle implications by Christiano et al. (2011). Goodfriend and McCallum (2007) contributes by providing an endogenous explanation for steady state differentials between lending and money market rates. In our paper, we introduce financial friction via the second type: When the default rate is taken into consideration, the interest rate of loans will increase as a risk premium.

Since default has a vital role in financial fragility, we follow recent papers on financial fragility (Goodhart et al., 2006; Tsomocos, 2003; Tsomocos and Zicchino, 2005) to model endogenous default. The inclusion of the possibility of default in general equilibrium models can be traced back to Shubik and Wilson (1977), Dubey and Geanakoplos (1992), and Dubey et al. (2005), allowing us to formally analyse default in models with and without uncertainty. In the Arrow-

Debreu model, an implicit assumption is that all agents honour their obligations, meaning there is no possibility of default. However, when using models such as strategic market games (Shapley and Shubik, 1977), the introduction of minimal institutions –e.g. money, credit and default– becomes a logical necessity. In particular, Shubik and Wilson (1977) allows agents to choose their repayment rates; this means that equilibrium becomes compatible with partial or complete abrogation of agents' contractual obligations. If agents are not accountable for their repayments, the result is very predictable; they will rationally choose not to repay their debts. In response, we are led to introduce default penalties that constrain agents' choices of repayment. If these default penalties are infinite then no one will borrow, and the model is reduced to the standard Arrow-Debreu model; however, if these penalties are zero no equilibrium can be established, since there will be unbounded credit demand and zero credit supply. Shubik and Wilson (1977) treats default by continuously allowing for partial default in equilibrium; thus they provide a useful framework as we encounter it in reality. We model default endogenously in this way and allow households and banks to choose their optimal repayment rates to maximise their utilities.

It is widely held that monetary policy can significantly influence the real economy, particularly in the short run. Indeed, much recent empirical research has confirmed the early finding of Friedman and Schwartz (1963) that monetary policy actions are followed by movements in real output that may last for two years or more (Romer and Romer, 1989; Bernanke and Blinder, 1992; Christiano, Eichenbaum, and Evans, 1994a, b). That said, there is less agreement on exactly how monetary policy exerts its influence. The traditional Keynesian portrayal of how a monetary policy is transmitted to the real economy is that policymakers use their leverage over short-term interest rates to affect the cost of capital, consequently affecting spending on durable goods such as fixed investments, housing, inventory, and consumer durables. In turn, changes in the level of aggregate demand influence the level of production. However, this story fails to explain the strength, timing and composition of monetary policy effects. These gaps have led to a new view of the monetary transmission mechanism, which emphasizes how asymmetric information and costly enforcement of contracts create agency problems in financial markets. Advocates of the credit channel state that monetary policy affects not only the general level of interest rates but also the size of the external finance premium – forming a wedge between the externally raised cost of funds and the opportunity cost of internal funds. Bernanke and Gertler (1995) suggest two mechanisms to explain the link between monetary policy actions and the external finance premium: the balance sheet channel (also known as the net worth channel) and the bank lending channel. Shifts in Fed policy affect the financial position of borrowers directly and indirectly through the balance sheet channel, also affecting the external finance premium by shifting the supply of intermediated credit (particularly loans by commercial banks) through the bank lending channel. Because of this additional policy effect on the external finance premium, the impact of monetary policy on the cost of borrowing – as well as on real spending and real activity – is magnified. In addition to the abovementioned channels, Mishkin (1995) identifies certain other transmission channels: the exchange rate channel and other asset price effects. Goodhart et al. (2006) also discusses the effect of expansionary monetary policy on financial fragility, with the default rate playing an essential role. Given an increase of liquidity in the economy, all prices increase according to the quantity theory of money proposition. Resultantly, the expected income of borrowers increases and so do their repayment rates. Furthermore, the volume of trade in the asset market is affected by the overall liquidity of the economy. In this way then, monetary policy interacts with asset markets and has an influence over asset prices (i.e. asset price inflation channel).

We focus on the second order effect of monetary policy in order to describe policy uncertainty and its effect on endogenous default. We will begin by defining what policy uncertainty is. Knight (1921) carefully distinguishes between economic risk and uncertainty. Unlike economic risk – wherein the outcomes are not known but are governed by probability distributions known at the outset – uncertainty represents a situation where not only the outcomes but even the probability models that govern them are unknown. According to Pastor and Veronesi (2012), there are two kinds of policy uncertainty. One is that given a policy, the effects on the current economy – both short-run and long-run – are uncertain. The other is called political uncertainty, meaning that we are uncertain about what action the government is going to take when solving social planning-related problems. In our case, we interpret the changes in the volatility of the innovations in the monetary policy as a representation of monetary uncertainty, following Fernández-Villaverde et al. (2011). The more volatile the policy is, the more uncertain it is. Our definition highlights that a more volatile policy may result in confusion about the intentions of the government.

In the second step, we consider the effect of policy uncertainty. There is some consensus among scholars in the field. Baker et al. (2012) believes policy uncertainty has a negative impact on recovery through an understanding that agents become more risk averse when faced with uncertainty: banks are more reluctant to lend money, and this issue becomes a top concern for small firms that decide to delay investment. Thus policy uncertainty poses a serious threat to business growth prospects, which holds back the pace of economic recovery. The paper also discusses the effect of uncertainty on investment and concludes that firms have an incentive to delay investment decisions because of the increased cost of finance and managerial risk aversion. Stokey (2013) reaches a similar conclusion: that uncertainty about a future tax change leads to a temporary reduction in investment, while investment recovers when the uncertainty is resolved. Bloom (2013) points out that uncertainty is countercyclical, rising in recessions and falling in booms. Shocks causing recession themselves induce uncertainty, which works as a propagation, so an amplification mechanism to explain why there is more uncertainty during the recessions can be understood. He also argues that policy uncertainty has a negative short-run impact on the economy while the impact is not clear in the long run due to more R&D expenditure. Similarly, Pastor and Veronesi (2013) argues that policy becomes more uncertain during recessions because policy makers want to do experiments. When the economy is doing well, politicians prefer to keep their current policy stances. But when the economy turns down, politicians are tempted to experiment as they attempt to revive growth. So policy uncertainty is the side effect of the search for policies to revive growth. Fernández-Villaverde et al. (2011) studies the effects of changes in uncertainty about future fiscal policy on aggregate economic activity and find that volatility shocks could have a sizable adverse effect on economic activity and inflation. Empirical works are done to investigate the impact of the policy uncertainty during a financial crisis. Baker et al. (2012) find evidence of the effect of low growth during the period 2008-2010, and evidence that policy uncertainty caused low growth after 2011. Through value at risk estimates, the study also demonstrates that innovation in policy uncertainty increased from 2006 to 2010, and points out that it is more suitable to treat policy uncertainty shocks as exogenous, because due to the forward looking nature of policy making, it is hard to empirically distinguish cause and effect between policy uncertainty and the economy. We treat monetary policy shock as an exogenous stochastic process following this argument.

There is some other research that focuses on the impact of uncertainty on asset pricing and concludes that asset price volatility is endogenously driven by economic uncertainty. Among these studies, the most important is Ulrich (2013), which mentions that monetary uncertainty is priced in the international equity markets. Pastor and Veronesi (2012) also figures out that announcement of policy changes will cause the price of the stock to fall, especially before a short and shallow downturn or if there is a great deal of uncertainty. There is a growing literature that analyses how other types of volatility shocks interact with aggregate variables. Nowadays, the majority of uncertainty comes from tax, spending and regulation, as in health care policy debates in the U.S. (Fernández-Villaverde et al., 2010). However, our paper is the first attempt to study the implication of monetary uncertainty on default as an equilibrium phenomena by estimating a monetary policy rule for the U.S. that allows for time-varying volatility. Our greatest aim is to find out a key transmission channel of monetary volatility shocks on default.

The rest of the paper is organised as follows. Section 2 presents our model, which forms the basis for our analysis. Section 3 lays down equilibrium analysis such as market clear conditions and optimality conditions. Section 4 explains calibration and discussion of our steady state. Section 5 discusses our results based on quantitative analysis and sections 6 concludes.

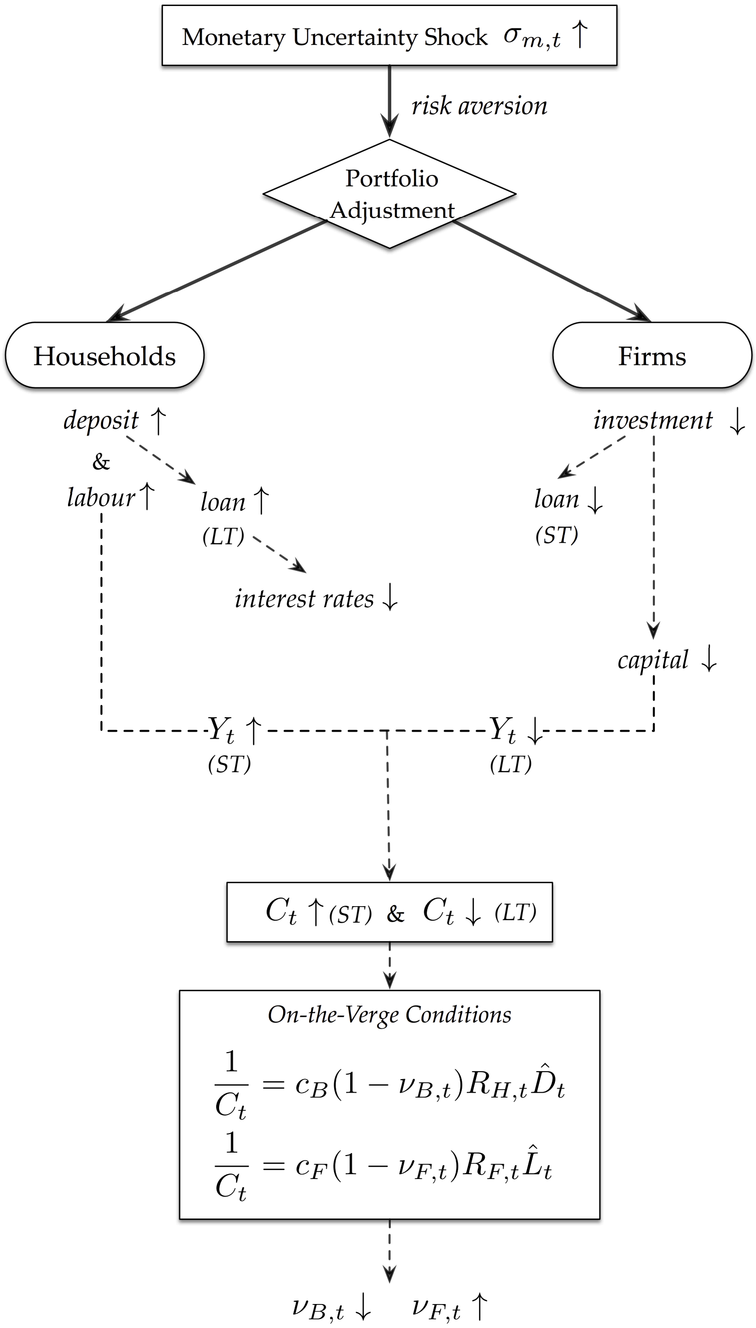

Figure 1: Transmission channel of monetary uncertainty

2 The model

2.1 Baseline model

We form our baseline model using a standard CIA constraints, detailed discussions of the model can be found, for instance, in Christiano (1991), Christiano and Eichenbaum (1992), Nason and Cogley (1994) and Schorfheide (2000).

The model economy consists of a representative household, a firm, and a financial intermediary, wihch is bank in this case. Output is produced according to a Cobb-Douglas production function,

$$Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha} \tag{1}$$

where K_t denotes the capital stock (predetermined at the beginning of period t), N_t is the labor input, A_t is the Total Factor Productivity (TFP), α is output elasticity of capital and $1-\alpha$ is output elasticity of labor.

The model economy is pertubated by three exogenous processes. Firstly, technology follows a stationary AR(1) process,

$$\begin{aligned} \ln A_t &= \rho_A \ln A_{t-1} + (1-\rho_A)\ln \bar{A} + \sigma_A \epsilon_{A,t} \qquad (2) \\ \epsilon_{A,t} &\sim i.i.d.\ N(0,1). \end{aligned}$$

ρ_A refers to the AR(1) coefficient of technology and $\bar{A}$ indicates the steady state of technology. The innovation $\epsilon_{A,t}$ follows a $N(0,1)$ process where σ_A denotes the standard deviation of innovations to $\ln A_t$.

The central bank lets the money stock M_t grow at rate $m_t = M_{t+1}/M_t$. m_t is a shifter to intertemporal money supply growth that follows AR(1) process:

$$\begin{aligned} \ln m_t &= \rho_m \ln m_{t-1} + (1-\rho_m)\ln \bar{m} + \chi\sigma_{m,t}\epsilon_{m,t} \qquad (3) \\ \epsilon_{m,t} &\sim i.i.d.\ N(0,1). \end{aligned}$$

Equation (3) can be interpreted as a simple monetary policy rule without feedbacks. $\chi\sigma_{m,t}\epsilon_{m,t}$

is the monetary policy shock. To put it more detail, the innovation $\epsilon_{m,t}$ to the monetary policy follows a $N(0,1)$ process that captures unexpected changes of the money growth rate due to normal policy making (Sims, 1982), and $\sigma_{m,t}$ measures standard deviation of innovation.

Then, we assumed the innovation $\chi\sigma_{m,t}\epsilon_{m,t}$ has a time-varying standard deviation $\chi\sigma_{m,t}$, i.e. stochastic volatility to proxy for monetary uncertainty. The principal novelty of this monetary policy is that, for the shifter m_t, the standard deviations $\chi\sigma_{m,t}$ of their innovations stochastically move period by period according to the autogressive processes (Fernandez-Villaverde, 2010):

$$\begin{aligned} \ln \sigma_{m,t} &= \rho_\sigma \ln \sigma_{m,t-1} + (1-\rho_\sigma) \ln \bar{\sigma}_m + \eta_\sigma \epsilon_{\sigma,t} \\ \epsilon_{\sigma,t} &\sim i.i.d.\ N(0,1). \end{aligned} \tag{4}$$

At the beginning of period t, the representative household inherits the entire money stock of the economy, Mt. The aggregate price level is denoted by P_t. In the standard CIA model, all decisions are made after, and therefore completely reflect, the current period's surprise change in money growth and technology. The household determines how much money D_t to deposit at the bank, while these deposits earn interest at the rate $R_{H,t}$. The bank receives household deposits and a monetary injection X_t from the central bank, which it lends to the firm at rate $R_{F,t}$.

The firm starts production and hires labor services from the household. After the firm produces its output, it uses the money borrowed from the bank to pay wages $W_t H_t$, where W_t is the nominal hourly wage and H_t is hours worked. The household's cash balance increases to $M_t - D_t + W_t H_t$. The CIA constraint implies that before a consumer can buy goods, they must pay for them in cash. The firm's net cash inflow is paid as dividend F_t to the household. Moreover, the household receives back its bank deposits inclusive of interest and the net cash inflow of the bank as dividend B_t.

Note that budget constraints for all agents are binding, because money is fiat and agents do not derive any utiliy from holding it. Thus, individuals do not hold any idle cash; instead they lend it out to someone who needs it.

Households

In period t, the household chooses consumption C_t, hours worked H_t, and non-negative deposits D_t to maximize the expected sum of discounted future utility. Resultantly, it solves the problem:

$$\max_{\{C_t, H_t, M_{t+1}, D_t\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1-\phi) \ln C_t + \phi \ln(1-H_t) \right\}$$
$$s.t.$$

$$P_t C_t \leq M_t - D_t + W_t H_t \tag{5}$$

$$0 \leq D_t \tag{6}$$

$$M_{t+1} = (M_t - D_t + W_t H_t - P_t C_t) + R_{H,t} D_t + F_t + B_t, \tag{7}$$

where β refers to the discount factor, ϕ measures the marginal rate of substitution between leisure and consumption, and $E_0(\cdot)$ is the expectation operator conditional on date 0.

The first constraint spells out the CIA constraint including wage revenues, the second the inability to borrow from the bank, and the third the intertemporal budget constraint emphasizing that households accumulate the money from total inflows made up of the money they receive from firms F_t and from banks B_t.

Firms

The firm chooses the next period's capital stock K_{t+1}, labor demand N_t, dividends F_t and loans L_t. Since households value a unit of nominal dividends in terms of the consumption it enables during period $t+1$, and firms and the financial intermediary are owned by households, date t nominal dividends are discounted by date $t+1$ marginal utility of consumption. Thus, the firm solves the problem:

$$\max_{\{F_t, K_{t+1}, N_t, L_t\}} \quad E_0 \sum_{t=0}^{\infty} \beta^{t+1} \frac{F_t}{C_{t+1} P_{t+1}}$$
$$s.t.$$

$$W_t N_t \le L_t \tag{8}$$

$$F_t = L_t + P_t \left[K_t^{\alpha} \left(A_t N_t \right)^{1-\alpha} - K_{t+1} + (1-\delta) K_t \right] - W_t N_t - L_t R_{F,t}. \tag{9}$$

The first constraint the firm faces reflects the fact that the firm finances its current period wage bill $W_t N_t$ by borrowing L_t. The second constraint says that the firm balances paying the household larger dividends or accumulating more capital. Thus this constraint links this decision with labor demand and loan demand using the Constant Return to Scale (CRS) production function $Y_t = K_t^{\alpha} \left(A_t N_t \right)^{1-\alpha}$ $(0 < \alpha < 1)$, the law of motion of capital defines gross investment $I_t = K_{t+1} - (1-\delta) K_t$ $(0 < \delta < 1)$, as well as goods market equilibrium $C_t + I_t = Y_t$.

Banks

The financial intermediary solves the trivial problem. The bank maximizes the expected infinite horizon discounted stream of dividends it pays to households:

$$\max_{\{B_t, L_t, D_t\}} \quad E_0 \sum_{t=1}^{\infty} \beta^{t+1} \frac{B_t}{C_{t+1} P_{t+1}}$$

$$s.t.$$

$$L_t \le X_t + D_t \tag{10}$$

$$B_t = D_t + R_{F,t} L_t - R_{H,t} D_t - L_t + X_t, \tag{11}$$

where $X_t = M_{t+1} - M_t$ is the monetary injection. Banks receive cash deposits D_t from households and a cash injection X_t, and then use these funds to disburse loans to the firms L_t, on which they make a net return of $R_{F,t}$. The second constraint simply defines the cash flow balances of the bank.

2.2 Extended model with default

However, the baseline model has neglected the existence of default, a significant issue in the recent crisis. The possibility of default on any debt obligations underscore the necessity of

CIA constraints. The interaction of liquidity and default justifies fiat money as the stipulated mean of the exchange. Otherwise, the mere presence of banks without possibility of default or any other financial friction in equilibrium may become a veil without affecting real trade and final equilibrium allocation. To expand on the baseline model, we introduce endogenous default via CIA constraints to better capture the fundamental aspect of liquidity and how it interacts with default to affect the real economy.

Following Shubik and Wilson (1997) and Dubey et al. (2005), we modelled the default that arises as an equilibrium phenomenon, because agents are allowed to choose what fraction to pay from their outstanding debt. The cost of default is modelled by a penalty that reduces utility, the non-pecuniary, instead of directly reducing an individual's ability to borrow after debtor defaults on a loan obligation.

Households

According to the discussion, the amount that the bank has to repay on its liability has to be adjusted for the bank's repayment rate $\nu_{B,t}$. In this sense, instead of receiving full amount $R_{H,t}D_t$, households receives $\nu_{B,t}R_{H,t}D_t$ indeed. Thus, in the model with endogenous default, equation (7) becomes

$$M_{t+1} = (M_t - D_t + W_t H_t - P_t C_t) + \nu_{B,t} R_{H,t} D_t + F_t + B_t. \tag{12}$$

Firms

The firms, as we mentioned in 2.1, are debtors of loans from banks. We introduce a variable $\nu_{F,t}$, the proportion firms actually pay back. Thus, the actual cash flow concerning about the interest paid to the banks become $\nu_{F,t}L_t R_{F,t}$ instead of $L_t R_{F,t}$. Additionally, since firms are allowed to defalut, which will be injurious to their reputation, their utility function will be reduced to some extent, and Non-pecuniary defalut penalty $\frac{c_F}{2}\frac{C_t}{M_t}\left[(1-\nu_{F,t})\, R_{F,t} L_t\right]^2$ describes this reputation cost in the firm's utility function. Date t reputation effects are discounted by date $t+1$ marginal utility of consumption just as F_t in the baseline model. So in the extended model, the utility function of firms in the baseline model changes to

$$\max_{\{F_t, K_{t+1}, N_t, L_t, \nu_{B,t}\}} E_0 \sum_{t=0}^{\infty} \frac{\beta^{t+1}}{C_{t+1} P_{t+1}} \left\{ F_t - \frac{c_F}{2} \frac{C_t}{M_t} \left[(1 - \nu_{F,t}) R_{F,t} L_t \right]^2 \right\}.$$

Accordingly, the budget constraint of firms (9) changes to

$$F_t = L_t + P_t \left[K_t^{\alpha} (A_t N_t)^{1-\alpha} - K_{t+1} + (1-\delta) K_t \right] - W_t N_t - \nu_{F,t} L_t R_{F,t}. \tag{13}$$

Banks

In the model with endogenous default, the changes of banks' utility function are similar to that of the firms. As we discussed above, $\nu_{B,t}$ is the repayment rate of banks, and

$$\frac{c_B}{2} \frac{C_t}{M_t} \left[(1 - \nu_{B,t}) R_{H,t} D_t \right]^2,$$

as we defined in the firm sector, measures the impairment of banks' reputation cost due to the default.

Thus, the utility function of the banks in baseline function changes to

$$\max_{\{B_t, L_t, D_t, \nu_{B,t}\}} E_0 \sum_{t=1}^{\infty} \frac{\beta^{t+1}}{C_{t+1} P_{t+1}} \left\{ B_t - \frac{c_B}{2} \frac{C_t}{M_t} \left[(1 - \nu_{B,t}) R_{H,t} D_t \right]^2 \right\}.$$

The banks only pay $\nu_{B,t} R_{H,t} D_t$ to the households, while banks only receive $\nu_{F,t} R_{F,t} L_t$ from the firm. As a result, the budget constraint of banks changes to

$$B_t = D_t + \nu_{F,t} R_{F,t} L_t - \nu_{B,t} R_{H,t} D_t - L_t + X_t. \tag{14}$$

3 Equilibrium analysis

3.1 Market clearing conditions

For the model economies, an equilibrium requires clearing in the goods, labor, credit, and money markets. All markets are assuemed to be perfectly competitive.

Baseline model

For the credit market, the equilibrium conditions are

$$R_{H,t} = R_{F,t} = R_t,$$

$$L_t = X_t + D_t,$$

$$B_t = D_t + R_{F,t} L_t - R_{H,t} D_t - L_t + X_t.$$

The equilibrium conditions indicate that the interest rates of deposits and loans are identical, and the dividend of banks is defined by the equilibrium market interest rate multiplied by the difference between the size of corporate loan and that of deposit, i.e. $B_t = R_t \left(L_t - D_t\right) = R_t X_t$.

For the labor market equilibrium, the labor supply equals the labor demand, i.e. $H_t = N_t$.

In equilibrium money market, budget constraints (5) and (8) are binding:

$$P_t C_t = M_t - D_t + W_t N_t,$$

$$W_t N_t = L_t.$$

Which implies the equilibrium condition in the money market,

$$P_t C_t = M_t - D_t + L_t = M_t + X_t = M_{t+1}.$$

The interpretation is that money demand, deliminated by nominal consumption demand $P_t C_t$ must be equated with money supply, which can be represented by current nominal balances M_t and monetary injections X_t.

In equilibrium, by combining budget constraints of households (7), firms (9) and banks (11), along with credit, labor and money market equilibrium conditions, we get equilibrium condition in the goods market:

$$C_t + K_{t+1} - (1-\delta) K_t = K_t^{\alpha} \left(A_t N_t\right)^{1-\alpha},$$

which indicates that output equals consumption plus investment, i.e, goods market clearing.

Extended model with default

There are some differences in the equilibrium conditions in the extended model when compared with baseline model.

For the credit market, the equilibrium conditions are

$$B_t = D_t + \nu_{F,t} R_{F,t} L_t - \nu_{B,t} R_{H,t} D_t - L_t + X_t,$$

$$L_t = X_t + D_t.$$

It implies $B_t = \nu_{F,t} R_{F,t} L_t - \nu_{B,t} R_{H,t} D_t$, just notice that since firms and banks have distinct repayment rates respectively, the equilibrium interest rates are not identical as in the baseline model. The credit market equilibrium conditions make it clear that the dividend of banks is defined by the difference between the size of corporate loan and that of deposit with corresponding interest rate, and is adjusted by payment rates.

For equilibrium conditions in the labor market, money market, as well as goods market, they are the same as in the baseline model.

3.2 Optimality conditions

We agree with proposition of money neutrality in the long run as RBC and New Keynesian literature suggests. While in the short run, our model obtains money non-neutrality equilibrium unlike the RBC models, where neutrality always holds. Furthermore, we consider the realization of short run non-neutrality equilirbium is driven by the postulated transaction technology, subsequent transactions and investment demand for money, where liquidity and default play the key role. While it is in stark contrast to the mechanism that New Keyneasian approach suggests to obtain short run non-neutrality, which is through real frictions such as monopolistic competition, asymmetric information, etc.

Baseline model

After solving for FOCs of households, we could get the following opitimality conditions

$$\frac{\phi C_t P_t}{(1-\phi)(1-H_t)} - W_t = 0, \tag{15}$$

$$E_t\left(\frac{\beta R_{H,t}}{C_{t+1}P_{t+1}}\right) - \frac{1}{C_t P_t} = 0. \tag{16}$$

The intratemporal labor market optimality condition depends on the structure of the credit market. Since the firm must finance its current peirod wage bill with borrowed funds, the credit market structure affects the labor demand of the firm. Combining with firm's borrowing constraint, $L_t = W_t N_t$ and labor market equilibrium $H_t = N_t$, the first equation, (15) becomes the intratemporal labor market optimality condition, which links labor supply, the marginal rate of substitution between leisure and consumption, and labor demand,

$$\frac{\phi C_t P_t}{(1-\phi)(1-N_t)} - \frac{L_t}{N_t} = 0. \tag{17}$$

The second equation (16) is the Euler equation that refers to the optimality condition in the credit market based on date t information. Intuitively, we suggest that the household's loss in current consumption due to increasing its deposits in the banks matches the discounted expected gain in the future consumption from that deposit.

For the firms, after combing FOCs and equilibrium conditions, we get another two optimality conditions:

$$E_t\left[\frac{P_t}{C_{t+1}P_{t+1}} - \frac{\beta P_{t+1}}{C_{t+2}P_{t+2}}\left(\alpha K_{t+1}^{\alpha-1}\left(A_{t+1}N_{t+1}\right)^{1-\alpha} + (1-\delta)\right)\right] = 0, \tag{18}$$

$$\frac{(1-\alpha)\, P_t Y_t}{L_t} - R_t = 0. \tag{19}$$

The first condition (18) is the optimality in the goods market, which defines the trade-off the economy faces in moving the consumption over time, and the intertemporal consumption trade-off is in terms of marginal utility one period ahead weighted by the purchasing power of money.

The second condition (19) identifies the equilibrium interest rate that clears the credit market and is determined by the borrowing decision of the firm. At the margin, the firm equates the increase in its nominal revenue generated by labor to the nominal cost of borrowing required to pay for the labor.

Extended model with default

The optimality conditions are all the same as in the baseline model, other than five equations:

$$E_t\left(\frac{\beta \nu_{B,t} R_{H,t}}{C_{t+1}P_{t+1}}\right) - \frac{1}{C_t P_t} = 0, \tag{20}$$

$$\frac{c_F C_t \left(1-\nu_{F,t}\right)^2 R_{F,t}^2 L_t W_t}{M_t} + \nu_{F,t} R_{F,t} W_t = \frac{\left(1-\alpha\right) P_t Y_t}{N_t}, \tag{21}$$

$$c_F \left(1-\nu_{F,t}\right) R_{F,t} \frac{L_t}{M_t} = \frac{1}{C_t}, \tag{22}$$

$$c_B \left(1-\nu_{B,t}\right) R_{H,t} \frac{D_t}{M_t} = \frac{1}{C_t}, \tag{23}$$

$$\frac{c_B C_t \left(1-\nu_{B,t}\right)^2 R_{H,t}^2 D_t}{M_t} = \left(\nu_{F,t} R_{F,t} - \nu_{B,t} R_{H,t}\right). \tag{24}$$

The first equation (20) is the Euler equation that refers to the optimality condition in the credit market based on date t information for the extended model with endogenous default. Households' per unit of deposit today in terms of marginal utility of consumption needs to generate equal future payments of banks adjusted for the repayment rate. In this sense, the equation indicates that the households' loss in current consumption from increasing its deposits in the bank matches the discouted expected gain, adjusted for default of banks, in the future consumption from the deposit.

Combining with firm's borrowing constraint, $L_t = W_t N_t$, the second equation, (21), becomes

$$\frac{c_F C_t \left[\left(1-\nu_{F,t}\right) R_{F,t} L_t\right]^2}{M_t N_t} + \frac{\nu_{F,t} R_{F,t} L_t}{N_t} = \frac{\left(1-\alpha\right) P_t Y_t}{N_t}.$$

The interpretation is that at the margin, the firm equates the increase in its nominal revenue generated by an extra unit of labor to the nomial cost for the unit of the labor, which consists of two components: one is the repayment adjusted for default the firms actually pay back to the banks for per labor while the other is the cost associated with the non-precuniary default penalty for per labor. Besides, firms can use the excess money to hire more labor to produce due to default. The borrowing interest rate for firms is thus determined by the borrowing decision of the firm according to this optimality condition.

The third equation (22) identifies the optimal defalut decision of the firms, that is, what proportion the firm decides to default. On the one hand, the firms will suffer from reputation loss due to default, on the other hand, firms are owned by households, they could comsume the defaulted amount of money to make up some utility. Therefore, the firm's optimal decision of default occurs when the marginal gain from default in terms of maginal utility of consumption equals the maginal loss from default, represented by the reputation loss. Accordingly, the same interpretation goes with the fourth equation, (23): the banks' optimal decision of default occurs when the marginal gain from default equals the marginal loss from default.

Transforming the last optimality condition, (24), we have

$$c_B C_t \left[(1 - \nu_{B,t}) \, R_{H,t} \frac{D_t}{M_t} \right]^2 + \frac{D_t}{M_t} \nu_{B,t} R_{H,t} = \frac{D_t}{M_t} \nu_{F,t} R_{F,t}.$$

The intuitive interpretation is that if banks lend what they borrowed, i.e. D_t from househols, to firms, the money banks earn from this transaction should compensate for the cost of it. One part is cost associated with the non-precuniary default penalty, the other part is what banks actually paid to households, including interests.

Proposition 1: Fisher Effect

For the baseline model, suppose that households comsume $C_t > 0$ and deposit $D_t > 0$ for $\forall t \in T$. It means that economy works in goods as well as deposit markets. Then in any short run equilibrium, we have

$$\log R_t \approx \log E_t \left(\frac{U'_{C_t}}{\beta U'_{C_{t+1}}} \right) + \log E_t \left(\pi_{t+1} \right).$$

For the extended model, banks choose their optimal repayment rate $\nu_{B,t}$, excluding the corner solution, $\nu_{B,t} \neq 0$. Then, together with active market conditions for goods and deposit, we have the following condition in any short run equilibrium,

$$\log R_{H,t} \approx \log E_t \left(\frac{U'_{C_t}}{\beta U'_{C_{t+1}}} \right) + \log E_t \left(\pi_{t+1} \right) + \log \frac{1}{\nu_{B,t}}.$$

In the baseline model, the nominal interest rate is approximately equal to the real interest rate plus risk premium such as inflation risk. In the extended model, additional risk premium is added: default risk which is represented by $\log \nu_{B,t}^{-1}$. The 'Fisher Effect' proposition explains that nominal price is linked to consumption; if nominal variables are affected, real variables are also affected allocationally.

Proof: in the baseline model, when the optimality condition in the credit market is satisfied, we obtain the Euler equation (16),

$$R_t = E_t\left(\frac{C_{t+1}P_{t+1}}{\beta C_t P_t}\right).$$

Similarly, in the extended model, we obtain the Euler equation in the credit market (20),

$$R_{H,t} = E_t\left(\frac{C_{t+1}P_{t+1}}{\beta \nu_{B,t} C_t P_t}\right).$$

Taking logarithm for both Euler equations, (16) and (20), we get two equations mentioned in this proposition. For the extended model, we can interpretate this as an extension of Fisher Effect by considering default risk as one kind of risk premium when determining norminal interest rate.

Proposition 2: Quantity Theory of Money

In the baseline model as well as the extended model, when money market is in equilibrium, CIA constraints involved are binding which implies the following equilibrium condition,

$$P_t C_t = M_{t+1}.$$

If the Quantity Theory of Money holds, the expression of the term $\frac{PY}{M}$ is constant.[1] However, when adopting CIA constraints in both models, we have

$$\frac{P_t Y_t}{M_{t+1}} = 1 + \frac{P_t I_t}{M_{t+1}}.$$

Therefore, the Quantity Theory of Money doesn't hold in the short run. The investment desicion is distorted by moneary policy and this distortion is transmitted into the real economy. The non-trivial role of money is thus confirmed.

Proof: as in Cooley and Hanson (1989), the CIA constraint always holds with equality. So just like what we have showed in equilirbium analysis, in equilibrium money market, two constraints (5) and (8) are binding: $P_t C_t = M_t - D_t + W_t N_t$ and $W_t N_t = L_t$. By inserting (8) into (5), we get the equilibrium condition in the money market, $P_t C_t = M_t - D_t + L_t = M_t + X_t = M_{t+1}$. Then, we combine this CIA constraint with aggregate resource constraint, $Y_t = C_t + I_t$, the expression of the term $\frac{PY}{M}$ is expressed as

[1]The Quantity Theory of Money is expressed as $MV = PY$, M refers to nominal money supply, P is price level, Y is real output and V is velocity of circulation of money. It is usually assumed that V is relatively constant, thus the Quantity Theory of Money claims that the growth rate of price level plus the growth rate of output is equal to the the growth rate of money supply.

$$\frac{P_tY_t}{M_{t+1}} = \frac{P_t\left(C_t + I_t\right)}{M_{t+1}} = \frac{P_tC_t}{M_{t+1}} + \frac{P_tI_t}{M_{t+1}} = 1 + \frac{P_tI_t}{M_{t+1}}.$$

Definition: No Arbitrage Conditions

Agents do not repay more than what they own, $\nu_{B,t}, \nu_{F,t} \leq 1$, and they are not rewarded for defaulting on their obligations $\nu_{B,t}, \nu_{F,t} \geq 0$. Consequently, endogenous default is compatible with the orderly function of the market economy.

Corollary 1: Money Non-neutrality

As money balances are held at the cost of foregone interest, positive interest rates reduce the efficiency of trade, so monetary policy is non-neutral.

Proposition 3: On-the-Verge Conditions

Suppose that 'No Arbitrage Conditions' and 'Market Clearing Conditions' for the credit market hold for $\forall t \in T$. Then, in any equilibrium, we obtain the detrended form of the equations:

$$\begin{aligned} U_{C_t} &= c_B\left(1-\nu_{B,t}\right)R_{H,t}\hat{D}_t, \\ U_{C_t} &= c_F\left(1-\nu_{F,t}\right)R_{F,t}\hat{L}_t. \end{aligned}$$

These conditions imply that the optimal amount of default is defined when the marginal utility of default equals the marginal disutility of it whenever firms or banks make a default desicion.

Proof: from the optimality conditions, (22) and (23) for firms and banks in the extended model we have

$$\frac{1}{C_t} = c_F\left(1-\nu_{F,t}\right)R_{F,t}\frac{L_t}{M_t} \text{ and } \frac{1}{C_t} = c_B\left(1-\nu_{B,t}\right)R_{H,t}\frac{D_t}{M_t}.$$

Then the detrended forms of the equations are

$$\frac{1}{C_t} = c_F\left(1-\nu_{F,t}\right)R_{F,t}\hat{L}_t \text{ and } \frac{1}{C_t} = c_B\left(1-\nu_{B,t}\right)R_{H,t}\hat{D}_t.$$

Since we specified the utility function as $\ln(\cdot)$ form, we rewrite the detrended forms of the equations as

$$U_{C_t} = c_B\left(1-\nu_{B,t}\right)R_{H,t}\hat{D}_t \text{ and } U_{C_t} = c_F\left(1-\nu_{F,t}\right)R_{F,t}\hat{L}_t.$$

Proposition 4: Relative Structure of Interest Rates

Suppose that banks receive deposits $D_t > 0$ from households at the cost of gross interest rate $R_{H,t} > 1$, while it lends loans $L_t > 0$ to the firms on which they make a net return of $R_{F,t} > 1$. Then in any short equilibrium, we can get the relationship of these two interest rates,

$$\nu_{F,t}R_{F,t} = R_{H,t}.$$

The condition implies that the only wedge between interest rate of loans and deposit rate is driven by repayment rate of firm. The lower the repayment rate is, the riskier the loans to firms become from bank's perspective. Thus, the wedge between these two interest rates enlarges.

Proof: in the extended model, when the optimality condition of the credit markets for firms, (24), is satisfied, we have

$$c_B C_t\left(1-\nu_{B,t}\right)^2 R_{H,t}^2\frac{D_t}{M_t} = \nu_{F,t}R_{F,t} - \nu_{B,t}R_{H,t}.$$

Also, from the optimality condition for banks in the extended model, (22), we have

$$\frac{1}{C_t} = c_B\left(1-\nu_{B,t}\right)R_{H,t}\frac{D_t}{M_t}.$$

Substituting C_t in (24) with that in (22) and rearranging the equation, we finally get $\nu_{F,t}R_{F,t} = R_{H,t}$.

Corollary 2: Order of Interest Rates

According to 'No Arbitrage Conditions' $(0 \leq \nu_F \leq 1)$, the order of interest rates can be confirmed: $R_{F,t} \geq R_{H,t}$.

3.3 Notes on endogenous default

Borrowers deliver on their promises considering punishment for their default decision. There are two types of default in modelling perspective that results in different penalties; default on secured debt results in the loss of collateral while default on unsecured debt generally brings pecuniary penalties (such as a search cost to find new loans) or non-pecuniary penalties (such as reputation loss). For discussion on the consequences of default, refer to Dubey et al. (2005). We introduce non-pecuniary default penalties (Tsomocos 2003) to capture the reputation cost due to default. Moreover, we adopt a quadratic form of non-pecuniary penalty and such a modeling approach allows for time-varying consumption levels and for a positive correlation between the repayment rate and consumption.

To model default penalty, it is reasonable to consider that more amount of default leads to more reputation loss. So we follow de Walque et al. (2010) , Tsomocos (2003) to assume a proportional relationship between the amount of default and reputation loss due to it, and the coefficient is denoted as $c_{F,t} > 0$. The reputation loss of firms, for example, can be thus described as $c_{F,t}\left(1-\nu_{F,t}\right)R_{F,t}\hat{L}_t$. However, the constant coefficients for default penalty implies no variances in the borrowing agent's intertemporal consumption, which strongly violates most of the dynamic. Thus, in our modelling approach, we allow time varying coefficient $c_{F,t}$ and assume coefficient for default penalty is a function of the state of the agent such as consumption level and amount of default to reflect the reality, e.g. for firms $c_{F,t} = \frac{c_F}{2}C_t\left(1-\nu_{F,t}\right)R_{F,t}\hat{L}_t$, where c_F is constant. As a result, the final forms of default penalty are $\frac{c_F}{2}C_t\left[\left(1-\nu_{F,t}\right)R_{F,t}\hat{L}_t\right]^2$ for firms and $\frac{c_B}{2}\hat{C}_t\left[\left(1-\nu_{B,t}\right)R_{H,t}\hat{D}_t\right]^2$ for banks respectively. Thus, the reputation cost of default is assumed to be proportional to the consumption level of households, who wholly owe firms and banks. In this case, the first-order conditions regarding borrower's repayment rate, (22) and (23), allow for a varying consumption level and a positive correlation between consumption and the repayment rate. Besides, this form suggests an increasing marginal cost of default, i.e. the punishment of default becomes more severe with increasing amount of default. In conclusion, the quadratic form of non-pecuniary penalty overcomes the problem raised by a simple linear form of default penalty.

4 Calibration

4.1 Empirical facts

To capture the default risk in each period in the real world, we first use the data series of nonperforming loans to total loans for all U.S. banks as shown in Figure 2. Nonperforming loans are defined as those past due 90 days or more and still accruing call, which is a proxy for our default measure: the higher the ratio is, the more default risk is in that period. We can see that the ratio increases at the beginning of a recession and starts to decrease at the end of the recession. Thus it shows that there exists a countercyclical risk premium in reality.

<Insert figure 2 here>

However, accounting data such as data on nonperforming loans have disadvantages because of (1) shifts in accounting practices both over time and between countries; (2) the continuing ability of bank management (and banks' auditors) to manipulate and smooth published accounts; and (3) the relatively long delays between the current effect of events on banks and their appearance in the accounts. Consequently, a rise in nonperforming loans tends to follow bank crises by many quarters.

The above analysis led us to switch to market data to better reflect the nature of recessions. Another alternative proxy for a risk premium or default measure to overcome the weakness of nonperforming loans is interest rate spread. We use the lending rate provided by the World Bank from 1980 to 2013, and use the 3-month treasury bond rate as a proxy for the risk-free rate in the same time range. Then, we denote the difference between the two as the interest rate spread. From Figure 2, we confirm that it is also countercyclical: high in a recession while low in a boom. Compared to nonperforming loans, the interest rate spread has almost reached the summit at the beginning of recession, which means the market data reacts faster than accounting data. This shows that in a recession, the risk of default is high, so that a higher risk premium is required to compensate and the interest rate spread enlarges, and vice versa.

4.2 Parameterisation

Given the complexity of the model, we solve the model numerically. We need to fix several parameter values to fit the model based on quarterly frequency and our guiding criterion in selecting them was to pick conventional values in the literature. We choose the discount factor $\beta = 0.99$ as a default choice. For the depreciation rate, $\delta = 0.025$ is set to induce the appropriate capital-output ratio (Fernández-Villaverde et al., 2010). We use $\alpha = 0.32$ for the capital share in the U.S. production function (Schmitt-Grohé and Uribe, 2003) and set the AR(1) coefficient of technology $\rho_A = 0.95$ with a standard deviation of innovation $\sigma_A = 0.007$ following Cooley and Prescott (1995).[2] Also, we set the value of total factor productivity in the steady state $\bar{A} = 1$. The AR(1) coefficient of the growth rate of monetary injections, $\rho_m = 0.6534$, as well as smoothing coefficients of policy uncertainty $\rho_\sigma = 0.964$, are estimated. (see the Appendix B). Nason and Cogley (1994) estimates a marginal rate of substitution of $\phi = 0.773$, and we use this estimate for our calibration. As we can see, all the above calibrated values are well within the range of values used in the macroeconomic literature. As for c_F and c_B, we apply reverse engineering method. We calibrate the steady state interest rates, $\bar{R}_H = 1.03$ and $\bar{R}_F = 1.04$.[3] After that, we calculate c_F and c_B, which correspond to the chosen interest rates in steady state. All of these implied parameters are reported in Table 1.

<Insert table 1 here>

4.3 Steady state and stability

To further examine the stability of the steady state, we compare the absolute values of the simultaneous equations. This implies the error of the calculation result, with the solutions specifying the minimum of all the endogenous variables. We then draw a graph –that is, Figure 3– for both baseline and extended models. If the error of the calculation result and the solutions of the minimum of all the endogenous variables are similar to each other, we

[2] The residual computed with measured real GDP is highly persistent, and the autocorrelations are quite consistent with a technology process that is a random walk. (Cooley and Prescott, 1995)

[3] $\bar{R}_F > \bar{R}_H$ (see the Proposition 4). Besides that, steady state interest rates are chosen so that the gaps between resulting c_F and c_B are not that big.

may suspect the validity of the result, since it will be hard for us to distinguish between the calculation error and the results. As the endogenous variables are quite far from the computation error, we can confirm that our steady state is quite stable.

<Insert figure 3 here>

5 Quantitative analysis

First, comparative statics is discussed to analyse the role of default in steady state. Then, we check the cyclical properties of our model through analysing impulse response functions (IRFs). Finally, we investigate the role of monetary uncertainty in default and clarify the transmission mechanism from monetary uncertainty to default via IRFs of uncertainty shock.

5.1 Steady state implication of default

The steady state of all endogenous variables is summarised in Table 2. The repayment rate of bank and firm satisfies 'No Arbitrage Conditions,' i.e. $\bar{v}_B$ and $\bar{v}_F$ are in the range of $[0, 1]$. Also, note that in the extended model, the steady state $\bar{R}_H$ is larger than that in the baseline model. This can be explained by the 'Fisher Effect' (Proposition 1): in the steady state, as stationarised price does not change over time, inflation risk does not exist, so the only difference is the default risk between the baseline and extended model, which is denoted by $\log \bar{v}_B^{-1}$. In the steady state in the extended model, firms and banks choose to default (partial default), and the results are not far from those of the baseline model.

<Insert table 2 here>

Furthermore, compared to the baseline model, the extended model presents some differences in the steady state. Thus, we can figure out the role of default by analysing such variations. First, when agents in the economy are allowed to default, banks and firms will choose the optimal repayment rate, $\bar{v}_B$ and $\bar{v}_F$, to maximise their utility; therefore the repayment rate for both banks and firms decreases. Meanwhile, the interest rate of loans $\bar{R}_H$ and $\bar{R}_F$ increases as the risk premium for taking into account default risk. Under these circumstances, households want to deposit more for higher interest, but banks cannot borrow as much as they want due to the high financing cost. Neither can firms. As a result, the amount of deposits $\bar{D}$ decreases, and accordingly firms borrow less in loans $\bar{L}$ from the banks. Second, after that, the default starts to have an impact on the real economy. The liquidity dries up to some extent, firms lose the opportunity to invest, and the production activity could be choked: output $\bar{Y}$ decreases, so as do labour and capital used for production $\bar{N}$ and $\bar{K}$. Third, the price is influenced. The wage per worker declines because the demand for labour decreases; The price $\bar{P}$ goes up since the supply of goods shrinks. Finally, agents consume less because of lower income and a higher price level, resulting in a decreased $\bar{C}$. In conclusion, the gap between the steady state in the extended model and that in the baseline model is reasonable and can be explained well economically. This suggests the validity of our calibration exercise in the extended model.

Since the choice of deposit rate and lending rate influences the coefficients of default penalty for firms and banks, we do the sensitivity analysis in the extended model. And all of them will have an impact on the default rates, 'On-the-Verge Conditions' (Proposition 3). The results are reported in Table 3.

<Insert table 3 here>

To do sensitivity analysis, we fix $\bar{R}_F$ and increase $\bar{R}_H$ within the limit of $\bar{R}_F$ to calculate the implied coefficient of default penalty for banks and firms, c_F and c_B, and repayment rates, $\bar{v}_B$ and $\bar{v}_F$.[4] We present the reasonable result that satisfies not only the 'No Arbitrage Conditions' ($0 < \bar{v}_j < 1$ for $j \in \{B, F\}$) but also the 'Fisher Effect' (Proposition 1, i.e. $\bar{R}_H \geq \bar{R}$). The analysis shows that there is a competing relationship between c_B and c_F, so between $\bar{v}_B$ and $\bar{v}_F$; When $\bar{R}_F$ is fixed and $\bar{R}_H$ increases, the coefficient of the default penalty for banks decreases while that for firms increases; the repayment rate of banks decreases while that of firms increases.

The explanation goes as follows: on the one hand, holding other factors equal, in the steady state, change in deposit rates reflects change in the default risk premium of banks. Thus, an increasing deposit rate leads to a decreasing repayment rate of banks $\bar{v}_B$ ('Fisher Effect,' Proposition 1). On the other hand, with an unchanged interest rate of loans to firms and an

[4]See Proposition 4, $\bar{R}_F$ should be greater than $\bar{R}_H$.

increasing deposit rate, the wedge between them enlarges, and it is reflected in the increasing repayment rate of banks $\bar{v}_F$ ('Relative Structure of Interest Rates,' Proposition 4). Finally, the marginal disutility of default should be equal to the marginal utility of default for both firms and banks. Since the marginal utility of default is the same for both, so is the marginal utility of default. We have $c_F\left(1-\nu_{F,t}\right)R_{F,t}\hat{L}_t = c_B\left(1-\nu_{B,t}\right)R_{H,t}\hat{D}_t$, with an increasing deposit rate $R_{H,t}$ and default rate of banks $\left(1-\nu_{B,t}\right)$ on the right side of the equation and only a decreasing default rate of firms $\left(1-\nu_{F,t}\right)$ on the left side. Here, the coefficient of the default penalty for banks c_B on the right hand should decline while that for firms c_F on the left hand should go up for the equation to hold (Proposition 3). Figure 4 presents this relationship more clearly.

<Insert figure 4 here>

5.2 Cyclical properties

We test for three hypotheses such as the financial accelerator effect (Bernanke et al., 1999), countercyclical risk premium (Gourio, 2012; Zhang, 2005; Storesletten et al., 2007) and pro-cyclical property of loans (Berger and Udell, 2004; Berger et al., 2001; Dell'Ariccia and Marquez, 2004) through analysis of IRFs of technological innovation. Then the non-trivial role of financial frictions, i.e. money and default, is confirmed by studying the IRFs of monetary policy shock.

We examine the IRFs of key macro-variables with respect to positive productivity shock. The results are reported in Figure 5. As expected, firstly, it directly increases output and the positive supply effect in the goods market induces the price level to go down. Secondly, a high level of productivity also improves the marginal product of labour, thus the equilibrium path of the wage rate increases in the short run. With the higher wage rate, there are two competing effects. One is the substitution effect; the higher wage rate increases the opportunity cost of leisure and encourages people to work more, driving labour inputs up. The other is the income effect; with a higher wage rate, the higher the income people receive. As they feel richer, they want to consume more. On the other hand, people seek to smooth consumption over time, which implies that increases in output and consumption will result in an increase in investment and therefore the capital stock. Third, with higher income, people tend to deposit more, and loans that banks lend to firms increase since banks have more deposits from households. As a result, with positive technology, the amount of loans to banks increases, and we can confirm the procyclical property of loans.[5] This generates a lower deposit rate and lending rate due to more liquidity in the system. Therefore, it encourages firms to increase investment. After that, increased capital stock and labour used for production and higher investment of firms all contribute to improve output again, working as a financial accelerator amplifying the productivity shock. Finally, with decreased marginal utility of default $\frac{1}{C_t}$ and increased loans, (L_t and D_t), repayment rates ($\nu_{B,t}$ and $\nu_{F,t}$) should increase even more for the 'On-the-Verge Conditions' (Proposition 3) to hold. Thus it is also shown that the endogenous firm repayment rate generates a countercyclical risk premium. As a result, with respect to productivity shock, our model generates a financial accelerator effect, countercyclical risk premium and procyclical property of loans.

<Insert figure 5 here>

We investigate the effects of expansionary monetary policy, as illustrated in the impulse responses to a money growth shock reported in Figure 6. We can observe that money growth shock has real effects on output and consumption as money is necessary to consume (i.e. CIA constraint). Thus, the non-trivial role of money is confirmed. Further, we observe that a temporary increase in the growth rate of money actually lowers output, labour input and consumption, and increases nominal interest rates. Moreover, repayment rates of firms and banks move in an opposite way, all of which are not particularly intuitive. The interpretation of these results is as follows. At first, the money growth shock increases the expected inflation rate, as the nominal interest rate is approximately equal to the real interest rate plus expected inflation ('Fisher Effect,' Proposition 1). Thus, the nominal interest rates $R_{H,t}$ and $R_{F,t}$ both rise. Our model predicts a sudden increase in the price level that results in high inflation in the first period. The more inflation there is, the less money people would like to hold, because inflation can be considered as a tax on the holders of money. However, since in

[5]As banks are naturally institutions pursuing profit, they will increase the supply of loans when the economy is good, and decrease the supply of loans when the economy is bad. This procyclical characteristic of banks' lending behaviour and theoertic hypothesis are proposed and verified by many scholars in various empirical works, such as Berger and Udell (2004), Berger et al. (2001) and Dell'Ariccia and Marquez (2004).

equilibrium they cannot hold less money than the central bank prints, people try to get away from money (consumption and deposits) and into leisure, so consumption, deposit and employment all go down immediately when the growth rate of money jumps up. This ends up reducing output again. In addition, investment increases temporarily following the increase in money, and therefore capital stock increases. Then, as there is a high level of working capital, the demand for labour services raises as well. Combined with a decreased labour supply, this results in a higher wage rate. After that, since there is more demand for money among firms (investment), loans to firms increase accordingly. The change of repayment rates can be explained by the 'On-the-Verge Conditions' (Proposition 3). As the increase in loans with interest $R_{F,t}\hat{L}_t$, which firms need to pay back, exceeds the increase in the marginal utility of default, the default rate of firms $(1-\nu_{F,t})$ needs to decrease for the 'On-the-Verge Conditions' to hold. Similarly, the increase in $R_{H,t}$ cannot make up for the reduction in $\hat{D}_t$, so that for the disutility of default to rise up to the level of increased utility of default, the default rate for banks $(1-\nu_{B,t})$ increases for the 'On-the-Verge Conditions' to hold.

<Insert figure 6 here>

5.3 Costs of the monetary uncertainty

We study the impact of temporal monetary uncertainty via IRFs. The results are reported in Figure 7. To figure out a transmission channel through which money uncertainty influences default, the analysis is divided into two stages. In the first stage, we confirm the transmission channel between monetary uncertainty and recession. As in the empirical works, we can observe that uncertainty is countercyclical in that more uncertainty is accompanied by recession. To explain this, Pastor and Veronesi (2012) argues that policy becomes more uncertain during recessions because policy makers want to experiment. When the economy turns down, politicians are tempted to do experiments as they attempt to revive growth. However, through our results, we could argue the reason for this phenomenon from the opposite direction as well: An increase in uncertainty itself could cause recession and thus works as a propagation and amplification mechanism of recession. The reasons are as follows. To start with, agents become more risk averse when faced with uncertainty. This change influences firms' decisions as well as households'. For firms, the uncertainty issue becomes a concern for them to delay investment in the short run as they are not sure what will happen to their investment in the future. Capital stock decreases as a result of decreased investment. Moreover, decreased demand for money (investment) leads to lower loans to the firms in the short run. As for households, they choose to work harder to offset the adverse effect that may be brought by more monetary uncertainty. The same logic applies when they choose to deposit more for precautionary purposes and consume less. Therefore, the wage rate decreases due to increased labour supply; the deposit rate falls because of more deposits from households (money supply); demand for goods declines as a result of lower consumption, and there are more loans in the long run because of increased deposits. After that, we could see the changing pattern of output. At first, it increases due to higher labour input. However, as time goes on, decreased capital dominates the role, and the output decreases. Besides, declining demand for goods also contributes to the decrease in the output. On the other hand, decreased output further results in lower consumption and investment, which works as a financial accelerator and causes depression. Thus in the first stage, monetary uncertainty leads to less output.

<Insert figure 7 here>

During the second stage, the society has entered into recession, and the default rates of banks and firms are influenced. According to the 'On-the-Verge Conditions' (Proposition 3), the marginal utility of default increases due to decreased consumption. However, $R_{F,t}$ and $\hat{L}_t$ both drop in the short run, driving the marginal disutility of default down. Thus, the default rate of firms $(1 - \nu_{F,t})$ needs to increase to equalise the marginal utility of default and the marginal disutility of it. Similarly, as $\hat{D}_t$ increases more than $R_{H,t}$ decreases, the default rate of banks $(1 - \nu_{B,t})$ decreases in the short run ('On-the-Verge Conditions,' Proposition 3). Banks increase the repayment rate at the cost of more debt. Therefore, in the second stage, the default rate of firms goes up and that of banks goes down. A countercyclical risk premium can be seen through the increased default rate of firms in the recession. Overall, we can confirm the role of monetary uncertainty in default, and clarify that the mechanism is that monetary uncertainty has an overall negative impact on the economy, especially it leads to less output by intensifying agents' degree of risk aversion, and finally drives up default rate of firms and decreases that of banks.

6 Concluding remarks

We employ a dynamic general equilibrium model with heterogeneous agents and financial frictions to analyse the impact of monetary uncertainty on economic activities, especially default. We interpret the changes in the volatility of the innovations in the monetary policy as a representative of monetary uncertainty following Andreasen et al. (2013). We find that increased monetary uncertainty can cause portfolio adjustment of agents due to their risk aversion. Households deposit more for precautionary reasons while investment from firms is delayed. The impact on the economy is that output drops; the default rate of firms climbs while that of banks drops at the cost of carrying more debt ('On-the-Verge Conditions,' Proposition 3). And the results are in line with current research on policy uncertainty, which holds that it has a negative effect on the economy generally, as argued by Baker et al. (2012), Stokey (2013), Fernández-Villaverde et al. (2011), and Bloom (2013).

Our paper is the first attempt to study the implications of monetary uncertainty on default as an equilibrium phenomenon. The existence of inherent links between monetary uncertainty and default may contribute towards current research that focuses on the effects of volatility shocks in several ways. Firstly, though there is a growing literature that analyses how various types of volatility shocks affect aggregate variables, this paper works as a supplement to the types of volatility shocks in current research by studying monetary uncertainty in a CIA model. Secondly, monetary uncertainty is considered in a framework of endogenous default, which serves as another innovation of this paper. Finally, this paper presents monetary policy implications for the government by revealing that destabilising effects of this type of policy are entirely avoidable if the policy maker adopts systematic "rules" about how to behave in any particular economic environment (Stokey, 2013).

Our results are generated under several assumptions. First, we have ignored the effects of heterogeneity at the level of banks. When faced with monetary uncertainty, different banks will respond differently according to their portfolio and risk aversion. In this sense, transmission channel of monetary uncertainty via interbank market is neglected. Second, to construct the monetary policy rule, we assume a simple case where money growth rate in the steady state never changes, while in reality there are shifts in the steady state of money growth rate corresponding to economic conditions. It's more reasonable to investigate more specific monetary policy rules such as the regime switching model. Finally, we only model the cost of default as a reputation loss, which merely refers to default on unsecured debt in reality. To model default in a broader way, introducing default on secured loans which leads to the loss of collateral can be viewed as another direction for future research.

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Appendices

Appendix A. List of detrended equations

The problem of non-stationarity comes from having stochastic trends in technology and money.The non-stationarity is clearly revealed when attempting to identify the steady state of the model and realizing it does not have one. It can be shown that when shocks are null nominal, variables grow with M_t. All real variables $q_t = [Y_t, C_t, I_t, K_{t+1}]$ have no growth trend in our model since we assume a stationarized technology process, and labor L_t is stationary as well since there is no population growth. Thus, detrending therefore involves the following operations (where hats over variables represent stationary variables): for nominal variables, $\hat{Q}_t = Q_t / M_t$, where $Q_t = [D_t, L_t, W_t, P_t]$ (Griffoli, 2007).

Moreover, note that variables take the time subscript of the period in which they are decided; thus, in the case of the capital stock, today's capital stock is a result of yesterday's decisions. So, with the law of motion for capital accumulation $K_{t+1} = I_t + (1-\delta) K_t$, we are actually working with $K_t = I_t + (1-\delta) K_{t-1}$.

We are finally working with the set of equations with detrended variables in the baseline model.

$$
\begin{aligned}
\ln A_t &= \rho_A \ln A_{t-1} + (1-\rho_A) \ln \bar{A} + \sigma_A \epsilon_{A,t} \\
\ln m_t &= \rho_m \ln m_{t-1} + (1-\rho_m) \ln \bar{m} + \chi \sigma_{m,t} \epsilon_{m,t} \\
\ln \sigma_{m,t} &= \rho_\sigma \ln \sigma_{m,t-1} + (1-\rho_\sigma) \ln \bar{\sigma}_m + \eta_\sigma \epsilon_{\sigma,t} \\
E_t \left(\frac{\hat{P}_t}{C_{t+1} \hat{P}_{t+1} m_t} \right) &= E_t \left(\frac{\beta \hat{P}_{t+1}}{C_{t+2} \hat{P}_{t+2} m_{t+1}} \left(\alpha \frac{Y_{t+1}}{K_t} + (1-\delta) \right) \right) \\
\hat{W}_t &= \frac{\hat{L}_t}{N_t} \\
\frac{\hat{L}_t}{N_t} &= \frac{\phi C_t \hat{P}_t}{(1-\phi)(1-N_t)}
\end{aligned}
$$

$$
\begin{aligned}
(1-\alpha)\hat{P}_t Y_t &= \hat{L}_t R_t \\
\frac{1}{R_t} &= \beta E_t\left(\frac{C_t \hat{P}_t}{C_{t+1}\hat{P}_{t+1} m_t}\right) \\
C_t + K_t &= (1-\delta) K_{t-1} + K_{t-1}^{\alpha}(A_t N_t)^{1-\alpha} \\
\hat{P}_t C_t &= m_t \\
\hat{L}_t &= m_t - 1 + \hat{D}_t \\
Y_t &= K_{t-1}^{\alpha}(A_t N_t)^{1-\alpha}
\end{aligned}
$$

Using the same method to stationarize variables in the extended model we are finally working with 15 equations charactering the equilibrium path of the model.

$$
\begin{aligned}
\ln A_t &= \rho_A \ln A_{t-1} + (1-\rho_A)\ln \bar{A} + \sigma_A \epsilon_{A,t} \\
\ln m_t &= \rho_m \ln m_{t-1} + (1-\rho_m)\ln \bar{m} + \chi \sigma_{m,t}\epsilon_{m,t} \\
\ln \sigma_{m,t} &= \rho_\sigma \ln \sigma_{m,t-1} + (1-\rho_\sigma)\ln \bar{\sigma}_m + \eta_\sigma \epsilon_{\sigma,t} \\
E_t\left(\frac{\hat{P}_t}{C_{t+1}\hat{P}_{t+1} m_t}\right) &= E_t\left(\frac{\beta \hat{P}_{t+1}}{C_{t+2}\hat{P}_{t+2} m_{t+1}}\left(\alpha\frac{Y_{t+1}}{K_t} + (1-\delta)\right)\right) \\
\hat{W}_t &= \frac{\hat{L}_t}{N_t} \\
\frac{\hat{L}_t}{N_t} &= \frac{\phi C_t \hat{P}_t}{(1-\phi)(1-N_t)} \\
(1-\alpha)\hat{P}_t Y_t &= c_F C_t\left[(1-\nu_{F,t}) R_{F,t}\hat{L}_t\right]^2 + \nu_{F,t} R_{F,t}\hat{L}_t \\
\frac{1}{R_{H,t}} &= \beta E_t\left(\frac{\nu_{B,t} C_t \hat{P}_t}{C_{t+1}\hat{P}_{t+1} m_t}\right) \\
C_t + K_{t+1} &= (1-\delta) K_t + K_t^{\alpha}(A_t N_t)^{1-\alpha} \\
\hat{P}_t C_t &= m_t \\
\hat{L}_t &= m_t - 1 + \hat{D}_t \\
Y_t &= K_{t-1}^{\alpha}(A_t N_t)^{1-\alpha} \\
\frac{1}{C_t} &= c_F (1-\nu_{F,t}) R_{F,t}\hat{L}_t \\
\frac{1}{C_t} &= c_B (1-\nu_{B,t}) R_{H,t}\hat{D}_t \\
\nu_{F,t} R_{F,t} - \nu_{B,t} R_{H,t} &= c_B C_t (1-\nu_{B,t})^2 R_{H,t}^2 \hat{D}_t
\end{aligned}
$$

Appendix B. Data source and description

In this paper, we estimate our model using time series data of the money stock M1 for the U.S. economy. M1 data series are quarterly data covering the period 1959:Q1-2013:Q4 with 219 observations in total. The data source for that is "Organization for Economic Co-operation and Development" (OECD). The detail of data selection, processing and description is given as follows.

M1 growth rate:

Since OECD provides the end-of-quarter money supply M1, we get the quarterly growth rate by using M1 in the one-time period forward divided by that in the current time period. After taking logarithm, we estimate the stochastic process describing monetary policy and then compare the data of logarithm of M1 growth rate with the estimates from the model,

$$\begin{aligned} \ln m_t &= \rho_m \ln m_{t-1} + (1-\rho_m)\ln \bar{m} + \chi \sigma_{m,t}\epsilon_{m,t} \\ \epsilon_{m,t} &\sim i.i.d.\ N(0,1). \end{aligned}$$

The evolution of moneytary policy is specified as that intertemporal money supply growth follows AR (1) process and $\chi\sigma_{m,t}\epsilon_{m,t}$ captures the monetary policy shock. The estimate result for monetary policy is reported in Table 5, the AR (1) coefficient and constant are statistically significant.

<Insert table 4 here>

Based on the estimate result, we set smoothing coefficients of monetary policy $\rho_m = 0.6534$. The standard deviation of the monetary policy shock is 0.0098. Then, to get the value of χ, the variance of the residual is written by $\chi^2 var(\sigma_{m,t}) = 0.0098^2$ as $\sigma_{m,t}$ and $\epsilon_{m,t}$ are assumed to be indepedent. Notice that $var(\sigma_{m,t}) = \exp\left(2\ln\bar{\sigma}_m + \frac{2\eta_\sigma^2}{1-\rho_\sigma^2}\right) - \exp\left(2\ln\bar{\sigma}_m + \frac{\eta_\sigma^2}{1-\rho_\sigma^2}\right)$. [6] Thus, we could deal with this equation in the end to solve out $\chi = 2.8046$ by plugging in the value

[6] For the monetary policy, as $\ln m_t$ and $\ln m_{t-1}$ follow the same ergodic distribution, we have $var(\ln m_t) = \frac{\chi^2}{1-\rho_\sigma^2} var(\sigma_{m,t})$ and $E(\ln m_t) = \ln\bar{m}$. Similarly, $\ln\sigma_{m,t}$ and $\ln\sigma_{m,t-1}$ follow the same ergodic distribution, we have $E(\ln\sigma_{m,t}) = \ln\bar{\sigma}_m$ and $var(\ln\sigma_{m,t}) = \dfrac{\eta_\sigma^2}{1-\rho_\sigma^2}$. Thus $\sigma_{m,t}$ follows log normal distribution, $\ln\sigma_{m,t} \sim N\left(\ln\bar{\sigma}_m, \frac{\eta_\sigma^2}{1-\rho_\sigma^2}\right)$.

of parameters. After that, we can easily calculate the value of constant money supply growth rate in steady state $\bar{m} = 1.0137$ by solving out the equation, $(1 - \rho_m) \ln \bar{m} = 0.0047$, and the result is $\bar{m} = 1.0137$.

<Insert figure 8 here>

<Insert figure 9 here>

Also, according to the Figure 8, the data and the model estimates move together and nearly simultaneously. In Figure 9, outliers occurred in the early 1980s and around 2008-2010 are reasonable; The United States entered recession in January 1980 and returned to growth six months later in July 1980.[7] Due to the unchanged unemployement rate, a second recession in July 1981 started (Bednarzik et al., 1982). Remaining impacts of energy crisis and contractionary monetary policy adopted by the Federal Reserve to combat double digit inflation serve as driving forces of 1980 recession (Iden et al., 1982; Feldstein, 1994). The downturn ended sixteen months later, in November 1982.[7] The economy entered a strong recovery and experienced a lengthy expansion through 1990 (Gardner, 1994). This cannot be captured by the model; 2008-2010 was the period when global financial crisis has been ongoing. Government and central bank responded with unprecedented fiscal stimulus, monetary policy expansion and institutional bailouts. The US Federal Reserve set up several rounds of quantitative easing to stimulate economy. Those were extreme situation, while for the other part, the majority of the data can be explained by the model pretty well.

Stochastic volatility of monetary policy:

We estimate the time-varying volatility of monetary policy from the residual of $\ln m_t$ using a GARCH (1,1) model:

$$\sigma^2_{m,t} = \alpha + \beta \sigma^2_{m,t-1} + \gamma \epsilon^2_{m,t-1}.$$

<Insert table 5 here>

[7] United States Business Cycle Expansions and Contractions. National Bureau of Economic Research. Retrieved 8 April 2011.

After GARCH (1,1) model generates value of stochastic volatility of monetary policy at each time period, we compare those values with the model estimates from stochastic process of the monetary uncertainty:

$$\begin{aligned} \ln \sigma_{m,t} &= \rho_\sigma \ln \sigma_{m,t-1} + (1-\rho_\sigma) \ln \bar{\sigma}_m + \eta_\sigma \epsilon_{\sigma,t} \\ \epsilon_{\sigma,t} &\sim i.i.d.\ N(0,1). \end{aligned}$$

To capture the policy uncertainty, we assume the stochastic volatility $\sigma_{m,t}$ of monetary policy is indexed by time; that is it stocastically move period by period according to the AR (1) process, where the idiosyncratic shock to uncertainty is specified as $\eta_\sigma \epsilon_{\sigma,t}$. The estimate results for monetary uncertainty is reported in Table 6.

<Insert table 6 here>

Based on the results, we set smoothing coefficients of monetary uncertainty $\rho_\sigma = 0.9634$, this indicates that stochastic volatility plays an important role and it is persistent. Similarly, η_σ and $\bar{\sigma}_m$ can be solved out. The value of standard deviation of monetary uncertainty shock η_σ is the standard deviation of residual from this regression since it is a constant: $\eta_\sigma = 0.0889$. And the standard deviation of monetary policy shock in the steady state $\bar{\sigma}_m$ can be obtained through the equation $(1-\rho_\sigma) \ln \bar{\sigma}_m = -0.1675$ and the result is $\bar{\sigma}_m = 0.0097$.

<Insert figure 10 here>

<Insert figure 11 here>

Also, AR (1) coefficient is significant and model estimate matches the data well, thus it can explain the majority of the data series. Outliers occur in the early 1980s, early 1990s, early 2000s and 2008-2010. Except for the recession in the 1980s and financial crisis from 2008 to 2010, early 1990s is a period of economic downturn affecting much of the world, and early 2000s was a decline in economic activity which mainly occurred in developed countries followed after dotcom bubble. All of these outliers can find their explanation in the real economy.

Appendix C. Solution method

We choose a third-order perturbation method to solve the model, not only for the reason that the higher order approximation increases the computational accuracy compared to the linear method, but also that volatility shock only enters as an indepedent term in the third-order approximation of the policy functions, so that the direct role of volatility can be explored. As shown in Fernandez-Villaverde et al. (2009), in the first-order approximation, only level shocks take effect while volatility shock doesn't show up. However, in the second-order approximation, volatility shock only appears in the form of cross product with other variables. It can only have an effect through the channel of other variables.

Furthermore, third-order approximation results in the differences between the deterministic steday state values and the mean of the ergodic distribution of the endogenous variables in the model. Thus IRFs starts at the mean of the ergodic distribution in the absence of shocks instead of at the beginning of steady state. To compute IRF of endogenous variables reported in this paper, we follow Koop (1996), Fernandez-Villaverde et al. (2009) and Cesa-Bianchi and Fermamdez-Corugedo (2014):

First, we compute the mean of ergodic distribution for each variable:

1. We draw a series of random shocks $\epsilon_t = (\epsilon_{A,t}, \epsilon_{M,t}, \epsilon_{\sigma,t})$ for 2096 periods and discard the first 2000 periods as a burn in.
2. We compute the mean of the ergodic distribution for each variable in our model based on the remaining periods.

Second, we compute impulse responses for the monetary uncertainty shock $\epsilon_{\sigma,t}$:

1. We do the simulation Y_t^1 starting from ergodic mean with $\epsilon_{M,t}$ having random values and all zeros for $\epsilon_{A,t}$ and $\epsilon_{\sigma,t}$.
2. We add one standard deviation of shock $\epsilon_{\sigma,t}$ at period 1 with keeping all the other shocks unchanged for remaining periods.
3. We do the simulation Y_t^2 starting from ergodic mean with newly added shock.
4. IRF is equal to $Y_t^2 - Y_t^1$.
5. We iterate the above procedure, (1-4), 50,000 times and calculate its average, which is our IRF.

Appendix D. Figures and Tables

Figures

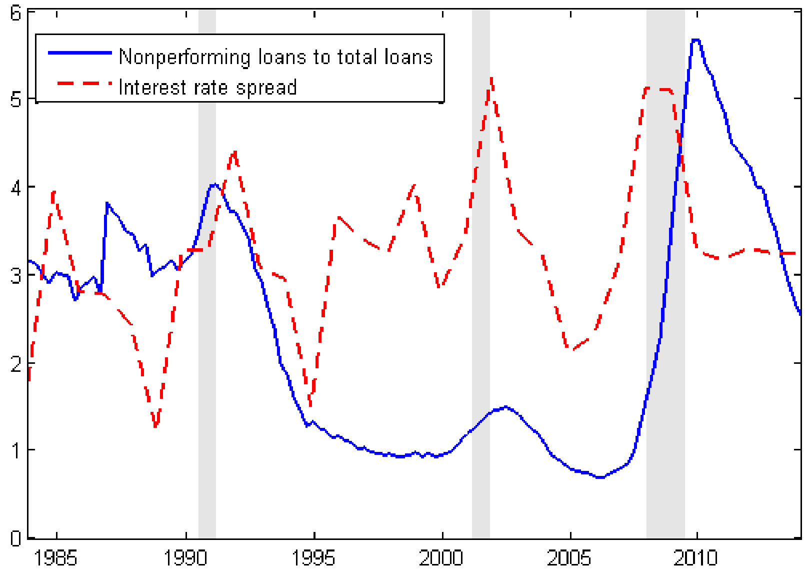


Figure 2: Default measures

Nonperforming loans are proxied for our default measure here. This series is constructed as a sum of total loan and lease finance receivables, nonaccrual call item RCFD1403 and total loan and lease finance receivables, past due 90 days or more and still accruing call item RCFD1407 in the total loans and leases, net of unearned income call item RCFD2122 for all U.S. banks, covering the period from 1984Q1 to 2014Q1. The data source is the Federal Financial Institutions Examination Council.

The interest spread is calculated as the difference between the lending rate and deposit rate from 1980 to 2013. The lending rate is provided by the World Bank, and the risk-free rate is proxied by the 3-month treasury bond rate. Shaded areas indicate U.S. recessions.

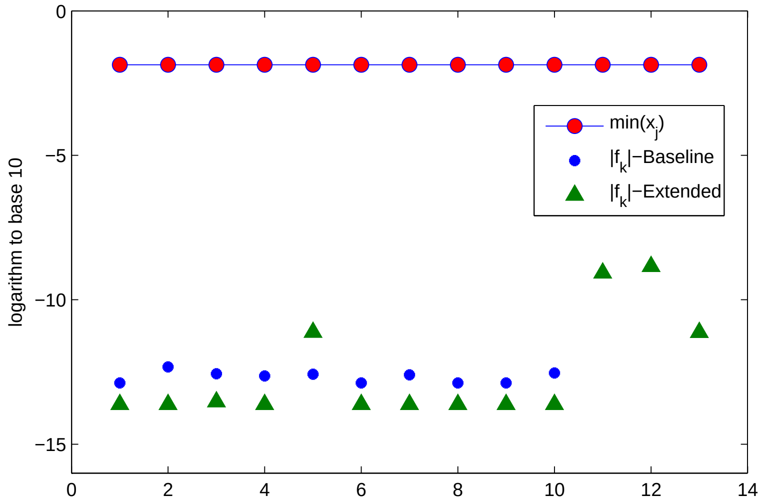
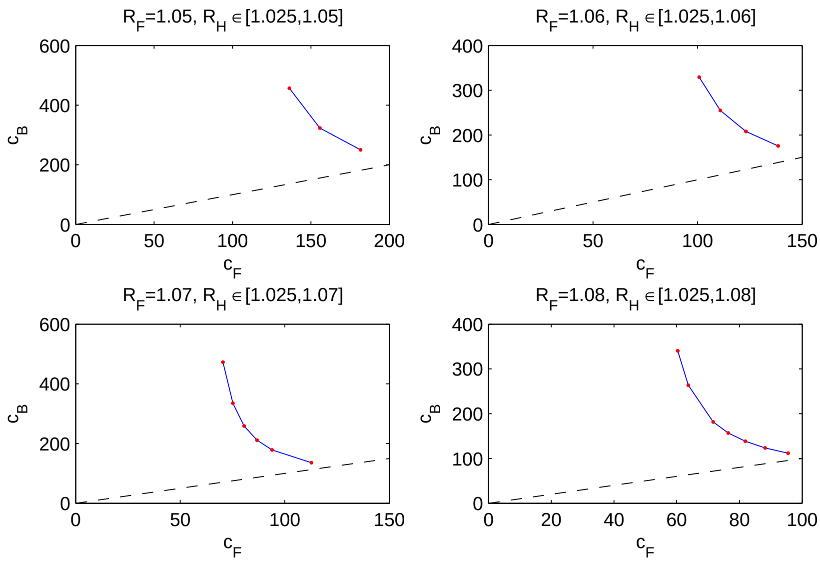
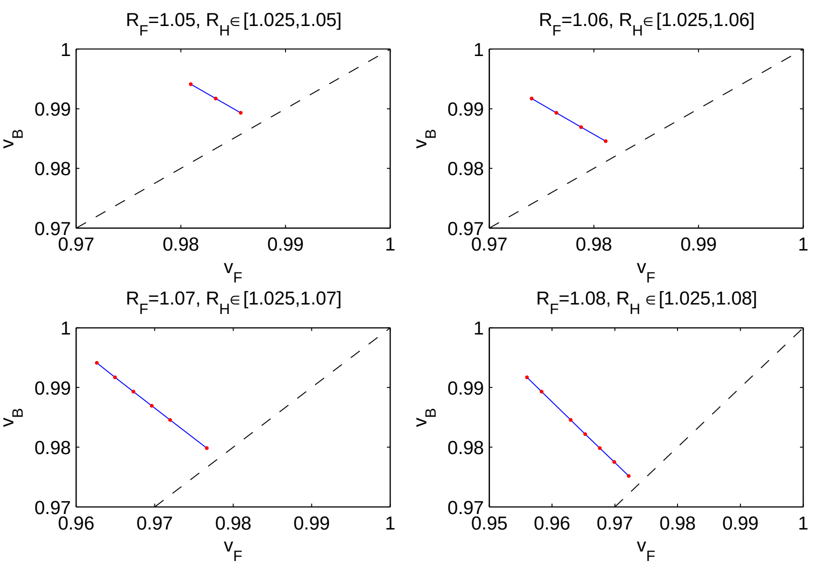


Figure 3: Calculation errors of the economy model

x_i represents the endogeneous variable in steady state for $i = 1, 2, \ldots, n$. $f_k = 0$ is the k^{th} simultaneous equation of steady state for $k = 1, 2, \ldots, n$. As $\forall\, |f_k| \leq 10^{-8}$ and $\min(x_i) \simeq 10^{-2}$, the steady state in this model economy is reasonably stable.



We fix interest rate for loans to firms, increase the deposit interest rate with a 0.0025 step, and draw the graph of the default penalty for bank and firms via the reverse engineering method. The dotted line is the one where $c_B = c_F$. From the graph, a non-linear negative relationship between c_B and c_F could be observed.



We fix the interest rate for loans to firms, increase the deposit interest rate with a 0.0025 step, and draw the graph of the repayment rate of banks and firms. The dotted line is the one where $\bar{v}_B = \bar{v}_F$. From the graph, an almost negative linear relationship between $\bar{v}_B$ and $\bar{v}_F$ can be observed.

Figure 4: Sensitivity analysis of $\bar{R}_H$ and $\bar{R}_F$

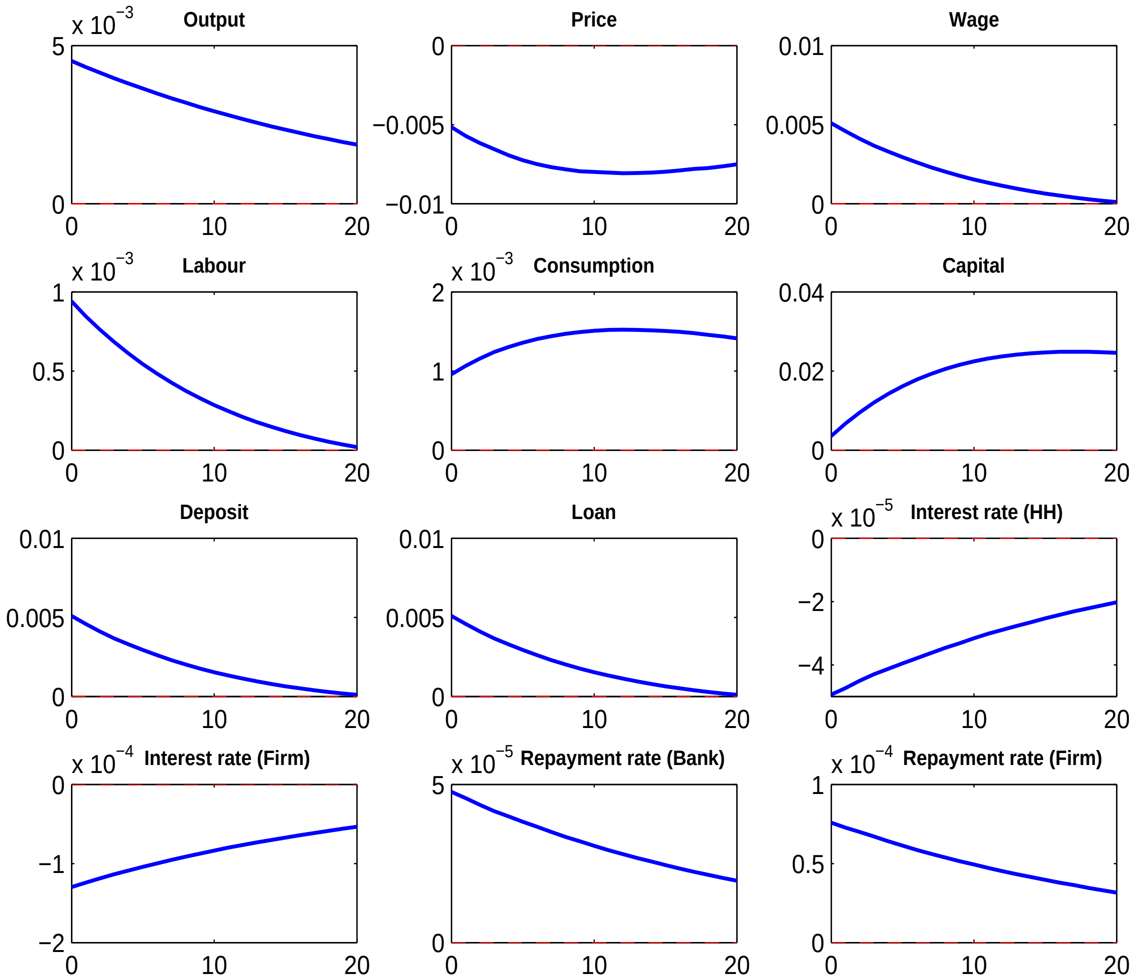


Figure 5: IRFs of positive productivity shock

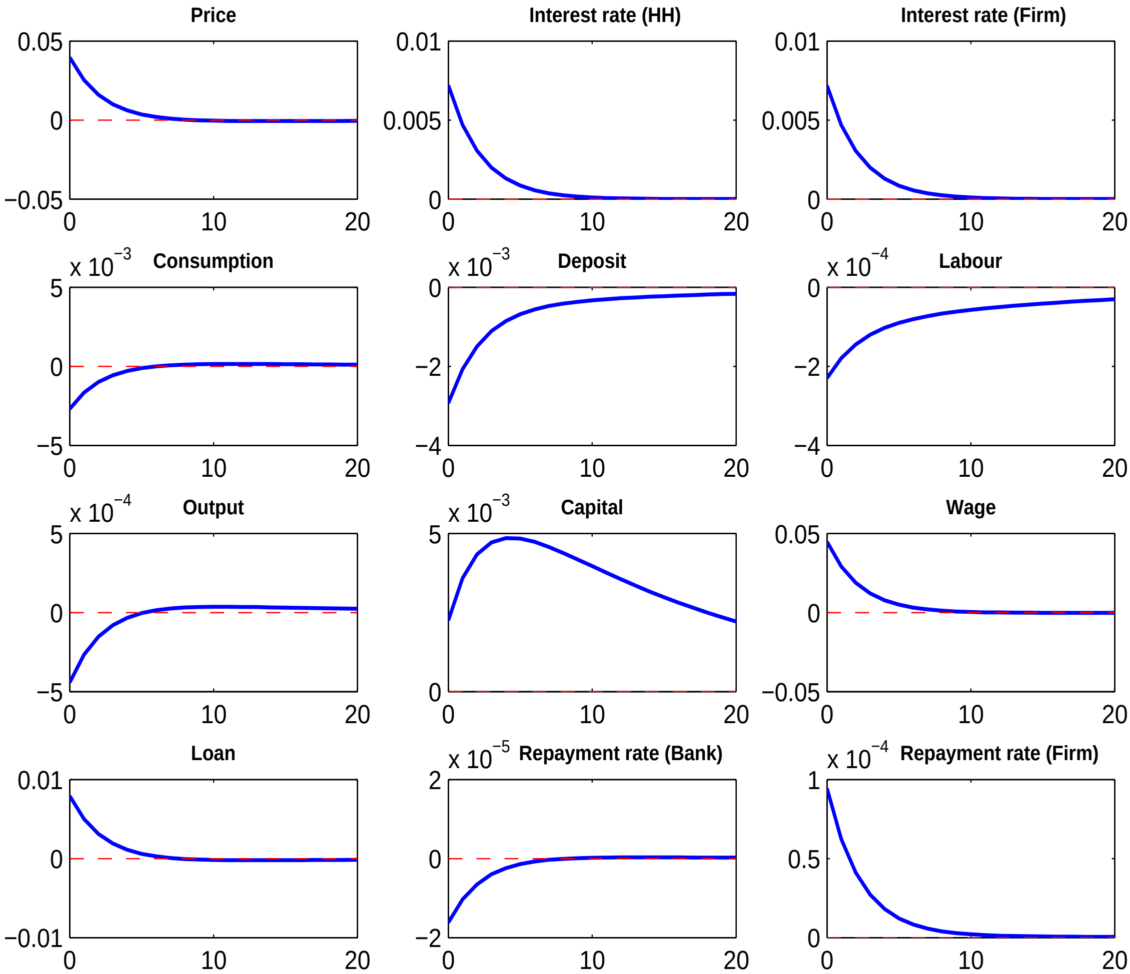


Figure 6: IRFs of positive money growth shock

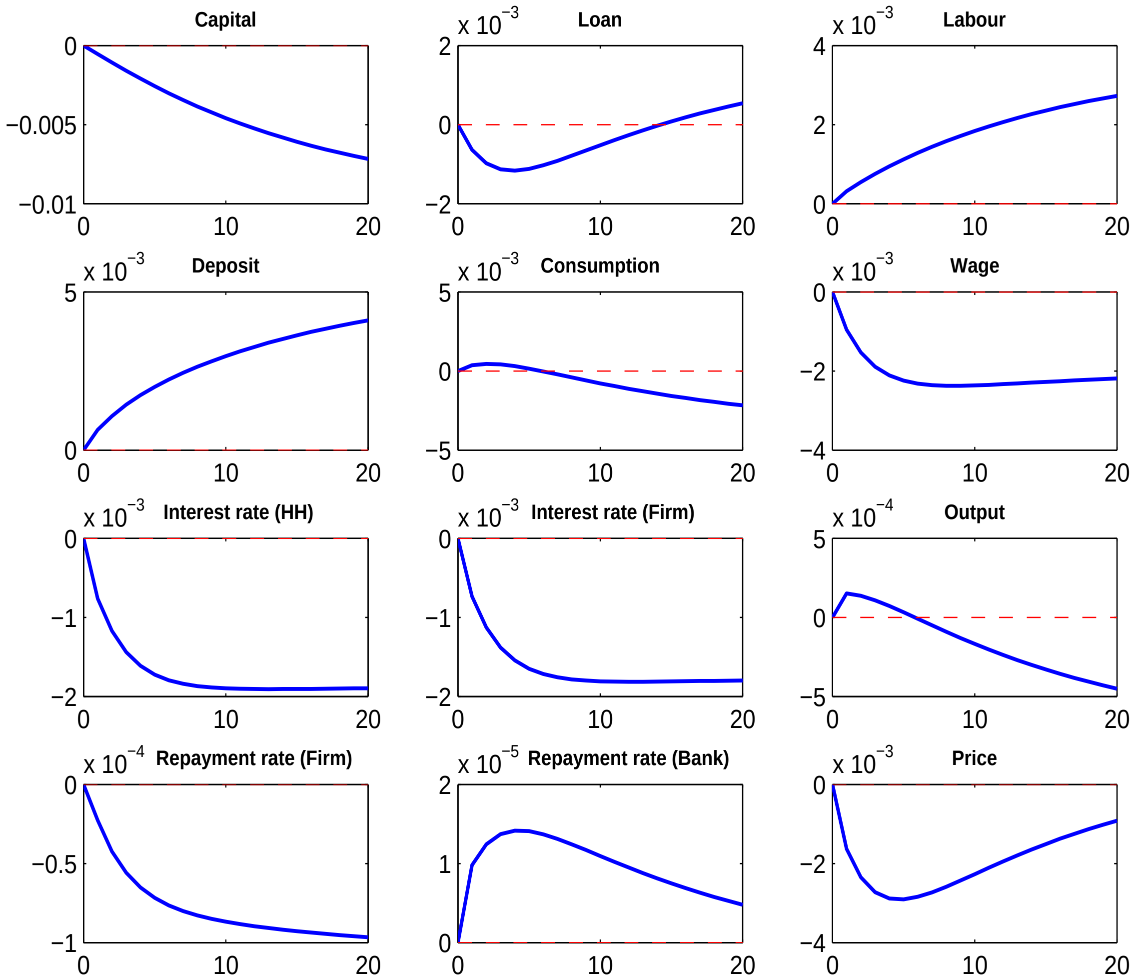


Figure 7: IRFs of monetary uncertainty shock

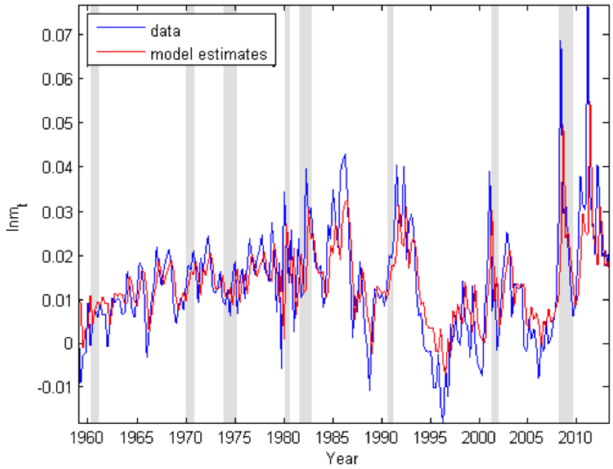


Figure 8: Monetary policy

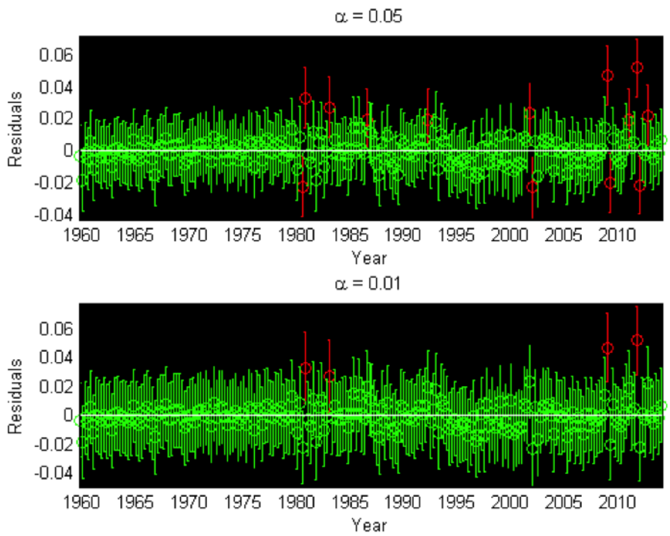


Figure 9: Residual plots of monetary policy

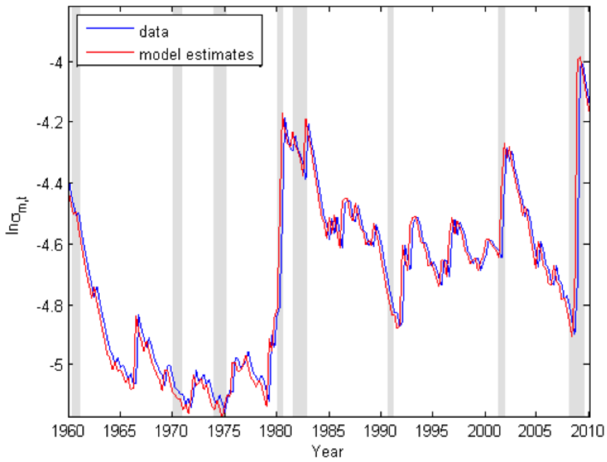


Figure 10: Estimate of monetary uncertainty

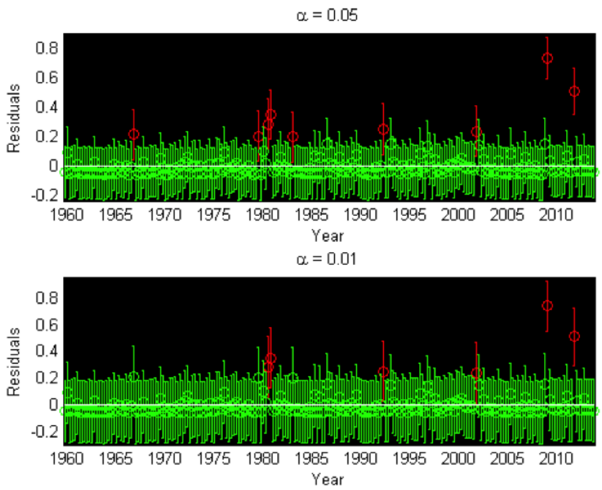


Figure 11: Residual plots of monetary uncertainty

Tables

Table 1: Implied parameters

Description	Parameter	Value
Output elastisity of capital	α	0.320
Discount factor	β	0.990
AR(1) coefficient of technology	ρ_A	0.950
Smoothing coefficient of monetary uncertainty	ρ_σ	0.964
AR(1) coefficient of money supply growth	ρ_m	0.653
MRS between leisure and consumption	ϕ	0.773
Depreciation rate	δ	0.025
Default penalty for banks	c_B	448.95
Default penalty for firms	c_F	267.75

Table 2: Steady state of the model

Description	Parameter	Baseline	Extended
Logarithm of monetary growth rate	$g\bar{M}$	0.0136	0.0136
Price of goods	$\bar{P}$	2.3012	2.3300
Consumption	$\bar{C}$	0.4405	0.4351
Wage per worker	$\bar{W}$	4.3239	4.3104
Capital used for production	$\bar{K}$	5.2014	5.1371
Interest rate for loans to firms	$\bar{R}_F$	1.0239	1.0400
Interest rate for deposit to banks	$\bar{R}_H$	1.0239	1.0300
Deposits from households	$\bar{D}$	0.8582	0.8448
Loans to firms	$\bar{L}$	0.8719	0.8585
Labor used for production	$\bar{N}$	0.2017	0.1992
Output	$\bar{Y}$	0.5705	0.5635
repayment rate of banks	$\bar{v}_B$	1	0.9941
repayment rate of firms	$\bar{v}_F$	1	0.9904

Table 3: Sensitivity analysis of $\bar{R}_F$ and $\bar{R}_B$

$\bar{R}_H$	c_B	c_F	$\bar{v}_B$	$\bar{v}_F$
		$\bar{R}_F = 1.04$		
1.025	2565.40	178.50	0.9990	0.9856
1.027	889.01	205.96	0.9970	0.9875
1.030	448.95	267.75	0.9941	0.9904
		$\bar{R}_F = 1.05$		
1.025	2610.5	108.96	0.9990	0.9762
1.030	456.83	136.20	0.9941	0.9810
1.035	250.32	181.60	0.9893	0.9857
		$\bar{R}_F = 1.06$		
1.030	464.78	92.37	0.9941	0.9717
1.035	254.67	110.84	0.9893	0.9764
1.040	175.39	138.55	0.9846	0.9811
		$\bar{R}_F = 1.07$		
1.030	472.79	70.50	0.9941	0.9626
1.035	259.06	80.53	0.9893	0.9673
1.040	178.41	93.95	0.9846	0.9720
		$\bar{R}_F = 1.08$		
1.030	480.88	57.32	0.9941	0.9537
1.040	181.46	71.65	0.9846	0.9630
1.045	138.38	81.89	0.9799	0.9676
1.050	111.83	95.54	0.9752	0.9722

Table 4: Estimate result for monetary policy

Description	Parameter	Standard error	p-Value
Constant	0.0047	0.0001	$< 10^{-4}$
AR(1) coefficient ρ_m	0.6534	0.0516	$< 10^{-4}$
RMSE	0.0098		
R-square	0.4260		

Table 5: GARCH (1,1) estimate result

Parameter	Value	Standard error
Constant	1.8831×10^{-6}	$< 10^{-4}$
GARCH(1)	0.8815	0.0322
ARCH(1)	0.1185	0.0338

Table 6: Estimate results for monetary uncertainty

Description	Parameter	Standard error	p-Value
Constant	-0.1675	0.0904	0.0653
AR(1) coefficient ρ_σ	0.9639	0.0193	$< 10^{-4}$
RMSE	0.0891		
R-square	0.921		